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Magnetohydrodynamic Rayleigh Problem with Hall Effect in a porous Plate

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ABSTRACT
This paper gives very significant analytical and numerical results to the magnetohydrodynamic flow version of the classical Rayleigh problem including Hall Effect in a porous plate. An exact solution of the MHD flow of incompressible, electrically conducting, viscous fluid past an uniformly accelerated and insulated infinite porous plate has been presented. Numerical values for the effects of the Hall parameter N, the Hartmann number M and the Porosity parameter P_0 on the velocity components u and v are tabulated and their profiles are shown graphically. The numerical results show that the velocity component u and v increases with the increase of N, decreases with the increase of P_0 and u decreases and v increases with the increase of M.
Keywords - MHD flow, Hall Effect, Viscous fluid, Uniformly accelerated plate, Porous plate

I. INTRODUCTION

Rossow[1] studied the MHD flow due to an impulsive start of an infinite plate without taking into account the induced magnetic field. With the induced magnetic field, it was solved by Nanda and Sundaram [2], Chang and Yen [3]. It was shown by Cowling[4] that when the strength of the magnetic field is very large, Ohm's law must be modified into Hall currents. MHD flow past a uniformly accelerated plate under a transverse magnetic field was studied by Gupta [5] and Pop[6]. Maleque and Sattar [7] investigated the steady laminar flow on a porous rotating disk with variable fluid properties taking Hall Effect into account. Hall effects on MHD flow past an accelerated plate was studied by R.K.Deka[8]. MHD Transient flow with Hall current past an accelerated horizontal porous plate in a rotating system was studied by Nazibuddin Ahmed, Jiwan Krishna Goswami, Dhruba Prasad Barua[9]. Haytham Sulieman, Naji A.Qatanani[10] studied the Magnetohydrodynamic Rayleigh problem with Hall effect.

MHD is important in solar physics, astrophysics, space plasma physics, and in laboratory plasma experiments. MHD and plasmas interactions are used to improve the performance of high speed aircraft and combustion inlets. The study of MHD viscous flows with Hall current has important engineering applications in problems of MHD generators, Hall accelerators as well as in flight Magnetohydrodynamics. The rotating flow of an electrically conducting fluid in the presence of magnetic field is encountered in cosmical and geophysical fluid dynamics.

In this study we have considered the effect of the Hall current on the magnetohydrodynamics flow version of the classical Rayleigh problem. Thus, an exact solution of the MHD flow of incompressible, electrically conducting and insulated infinite porous plate has been presented. Numerical results for the effects of the Hall parameter **N**, the porosity parameter P_0 and the Hartmann number **M** on the velocity components **u** and **v** are tabulated and their profiles are shown graphically.

II. FORMULATION OF THE PROBLEM

Consider the flow of an incompressible, electrically conducting, viscous fluid past an infinite and insulated flat porous plate occupying the plane y = 0. Let the positive direction of x –axis be chosen along the plate in the direction of the flow and the y – axis is normal to it. A uniform magnetic field H_0 is applied in the direction of the y – axis.

The physical configuration and nature of the flow suggest the velocity vector $\vec{q}$, magnetic induction vector $\vec{H}$, electro static field $\vec{E}$ and Pressure P as

$$\left.\begin{aligned} \vec{q} &= (u, 0, v), \\ \vec{H} &= (H_x,\ H_0, H_z) \\ \vec{E} &= (E_x, 0, E_z) \\ P &= \text{Constant} \end{aligned}\right\} \qquad (2.1)$$

The equations governing the unsteady flow and Maxwell's equations are:

Equation of continuity: $\nabla.\vec{q} = 0$ (2.2)

Equation of motion:

$$\frac{\partial \vec{q}}{\partial t} + (\vec{q}.\nabla)\vec{q} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\ \vec{q} + \frac{1}{\rho}\ (\vec{J}\times\vec{H}) - \frac{\mu\,\vec{q}}{\rho\,k} \qquad (2.3)$$

Equation for current: $\nabla \times \vec{H} = \mu\,\vec{J}$ (2.4)

Faraday's Law: $\nabla \times \vec{E} = -\frac{\partial \vec{H}}{\partial t}$ (2.5)

$\nabla \cdot \vec{H} = 0$ (2.6)

The generalized Ohm's law, neglecting ion– slip effect but taking hall current is,

$$\frac{\vec{J}}{\sigma} = (E + \vec{q} \times H) - \frac{(J \times H)}{ne} \quad (2.7)$$

where $\sigma = \frac{e^2 \tau n}{m}$ (is the electrical conductivity).

Here $\vec{J}$ is the current density, t is the time, ρ is the density, ν is the kinematic viscosity, μ is the magnetic permeability, e is the electric charge, m is the mass of an electron, n is the electron number density, and is the mean collision time.

The Lorentz force per unit volume is

$\vec{J} \times \vec{H} = [-J_z H_0, J_z H_x - J_x H_z, J_x H_0]$ (2.8)

$\vec{q} \times \vec{H} = [-vH_0, v H_x - u H_z, u H_0]$ (2.9)

where $\vec{J} = [J_x, 0, J_z]$

with $J_x = \frac{\sigma}{1+\tau^2\omega^2}[E_x - vH_0 + \omega\tau(E_z + uH_0)]$ (2.10)

$J_z = \frac{\sigma}{1+\tau^2\omega^2}[E_z + uH_0 - \omega\tau(E_x - vH_0)]$ (2.11)

and $\omega = \frac{eH_0}{m}$ (is the electron Larmor frequency)

The initial and boundary conditions are:

$$\left.\begin{array}{l} t \le 0: u = 0, \quad v = 0 \text{ for } y \ge 0 \\ t > 0: u = U_0, \quad v = 0 \text{ for } y = 0 \\ u \to 0: \quad v = 0 \quad \text{as } y \to \infty \\ H_x \to 0, \quad H_y = H_0, \quad H_z \to 0 \text{ as } y \to \infty \end{array}\right\} \quad (2.12)$$

At infinity, the magnetic induction is uniform with components (0, H, 0), and hence the current density in (2.4) vanishes. And since the free stream is at rest, it follows from generalized Ohm's law that E = 0 as y→∞. Assuming small magnetic Reynolds number for the flow, the induced magnetic field is neglected in comparison to the applied constant field H_0.

On introducing the non– dimensional quantities:

$$y^* = \frac{U_0 y}{\nu},\ u^* = \frac{u}{U_0},\ v^* = \frac{v}{U_0},\ t^* = \frac{U_0^2 t}{\nu} \quad (2.13)$$

Consequently, the equation of motion (2.3) in component term becomes (dropping the stars):

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \frac{M^2}{1+N^2}(u + N v) - P_0 u \quad (2.14)$$

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial y^2} + \frac{M^2}{1+N^2}(N u - v) - P_0 v \quad (2.15)$$

where $M^2 = \frac{\sigma H_0^2 \nu}{\rho U_0^2}$ is the Hartmann number,

$N = \omega\tau$ is the Hall parameter,

$P_0 = \frac{\nu^2}{k U_0^2}$ is the Porosity parameter and u and v are the velocity components in the x and y direction respectively.

The initial and boundary conditions become:

$$\left.\begin{array}{l} u(0,y) = v(0,y) = 0 \\ u(t,0) = 1, v(t,0) = 0 \\ u(t,y) \text{ and } v(t,y) \to 0 \text{ as } y \to \infty \end{array}\right\} \quad (2.16)$$

Now, multiplying both sides of equations (2.14) and (2.15) by e^{-st} and integrate from 0 to ∞ with respect to t, we get

$$\frac{d^2\hat{u}}{dy^2} - \left(\frac{M^2}{1+N^2} + s + P_0\right)\hat{u} = \frac{NM^2}{1+N^2}\hat{v} \quad (2.17)$$

$$\frac{d^2\hat{v}}{dy^2} - \left(\frac{M^2}{1+N^2} + s + P_0\right)\hat{v} = -\frac{NM^2}{1+N^2}\hat{u} \quad (2.18)$$

where $\hat{u}(s, y) = \int_0^\infty e^{-st} u(t,y)\,dt = L\{u(t)\}$

$\hat{v}(s,y) = \int_0^\infty e^{-st} v(t,y)\,dt = L\{v(t)\}$.

By introducing the complex function $\hat{q} = \hat{u} + i\hat{v}$, the equations (2.17) & (2.18) can be combined into the single equation

$$\frac{d^2\hat{q}}{dy^2} - \left\{\frac{M^2}{1+N^2}(1 - iN) + s + P_0\right\}\hat{q} = 0. \quad (2.19)$$

III. ANALYTICAL SOLUTION

By introducing the complex function $\hat{q} = \hat{u} + i\hat{v}$, then the equations (2.14) and (2.15) becomes,

$$\frac{\partial \hat{q}}{\partial t} = \frac{\partial^2 \hat{q}}{\partial y^2} - \left\{\left(\frac{M^2}{1+N^2}\right)(1 - iN) + P_0\right\}\hat{q} \quad (3.1)$$

The initial and boundary conditions take the form

$$\left.\begin{array}{l} \hat{q}(0,y) = 0, \ \hat{q}(t,0) = 1 \\ \hat{q}(t,y) \to 0 \text{ as } y \to \infty. \end{array}\right\} \quad (3.2)$$

Using the abbreviation $\alpha = -\left\{\left(\frac{M^2}{1+N^2}\right)(1 - iN) + P_0\right\}$, equation (3.1) can be written as

$$\frac{\partial \hat{q}}{\partial t} = \frac{\partial^2 \hat{q}}{\partial y^2} + \alpha \hat{q} \quad (3.3)$$

Let $\Phi(t, y) = e^{-\alpha t} q(t, y)$ (3.4)

From equations (3.2) and (3.3), we conclude that

$$\left.\begin{array}{l} \Phi(0,y) = 0, \Phi(t,0) = e^{-\alpha t} \\ \Phi(t,y) \to 0 \text{ as } y \to \infty \end{array}\right\} \quad (3.5)$$

To solve (3.3) and (3.4) subject to the initial and boundary conditions (3.5), we apply the Laplace Transform technique and obtain the solution as

$$q(t, y) = e^{at}\cos bt\ \mathrm{erfc}\left(\frac{y}{2\sqrt{t}}\right) - \int_0^t e^{a\tau}\,\mathrm{erfc}\left(\frac{y}{2\sqrt{\tau}}\right)[a \cos b\tau - b \sin b\tau]d\tau + i\left\{e^{at}\sin bt\ \mathrm{erfc}\left(\frac{y}{2\sqrt{t}}\right) - \int_0^t e^{a\tau}\,\mathrm{erfc}\left(\frac{y}{2\sqrt{\tau}}\right)[a \sin b\tau + b \cos b\tau]\right\}d\tau \quad (3.6)$$

where $q(t,y) = u + iv$, $\alpha = a + ib$,

$a = -\left(\frac{M^2}{1+N^2}\right) - P_0$, $b = \frac{M^2 N}{1+N^2}$.

Consider the following BVP

$$\frac{d^2\hat{q}}{dy^2} - \omega\hat{q} = 0 \quad (3.7)$$

where $\omega = \left(\frac{M^2}{1+N^2} + s + P_0\right) - i\frac{NM^2}{1+N^2}$

with $\hat{q}(0,s) = \frac{1}{s}$, $\hat{q}(\infty,s) = 0$. (3.8)

To ensure that the Laplace Transforms are well – defined, it should be assumed that s >0. This implies that

$\mathrm{Re}(\omega) = \left(\frac{M^2}{1+N^2} + s + P_0\right) > 0$.

Hence there exists η in the complex number such that $\eta^2 = \omega$ with Re (η) < 0.

The solution of (3.7) is

$$\hat{q}(y,s) = C_1 e^{\eta y} + C_2 e^{-\eta y} \quad (3.9)$$

By substituting the boundary conditions, (3.9) becomes

$$\hat{q}(y, s) = \frac{e^{\eta y}}{s} \quad (3.10)$$

(3.10) satisfies the above BVP.

Now, for y = 0, $\hat{q}(0,s) = \frac{1}{s} = \int_0^\infty e^{-st}\ dt$

$= \int_0^\infty e^{-st}\ (1 + i\ 0)\ dt\ \ \Rightarrow \int_0^\infty e^{-st}\ (u + iv)dt$

$= \int_0^\infty e^{-st}\ (1 + i\ 0)\ dt$

Thus, u (0, t) = 1 and v (0, t) = 0 for all t.

By inverse Laplace transform

$$q\ (y,\ t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \hat{q}(y, s)\ e^{st}\, ds \qquad (3.11)$$

where $\gamma > 0$ is chosen so that all the singularities of $\hat{q}(y, s)$ are to the left of γ. (3.11) is over the vertical line $z = \gamma$ in the complex plane.We can choose $\gamma > 0$ and let $\gamma = 0.25$.We will define q strictly as a function of t using Mathematics NIntegrate command.We will approximate the integral (3.11) by integrating from 0.25 – 500i to 0.25 + 500i.

The effect of the Hall parameter N, the Hartmann number M and the porosity parameter P_0 in the velocity components u and v is illustrated in the following figures.

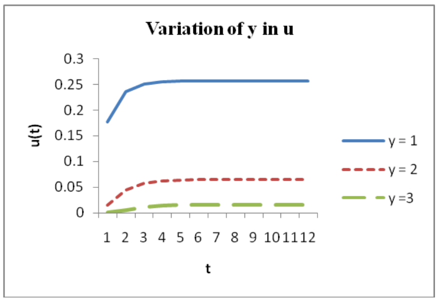

Figure 1

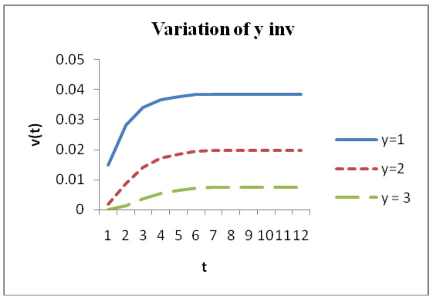

Figure 2

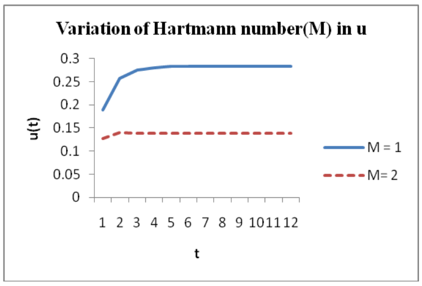

Figure 3

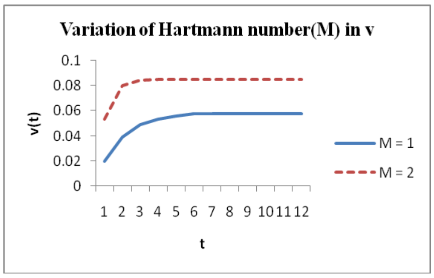

Figure 4

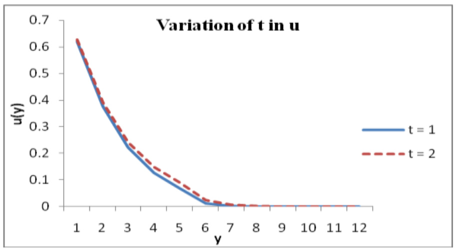

Figure 5

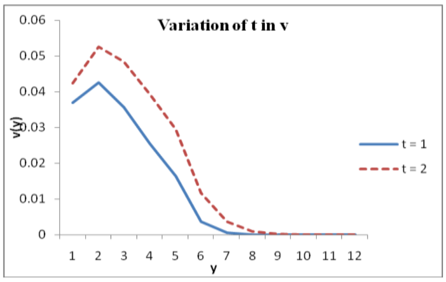

Figure 6

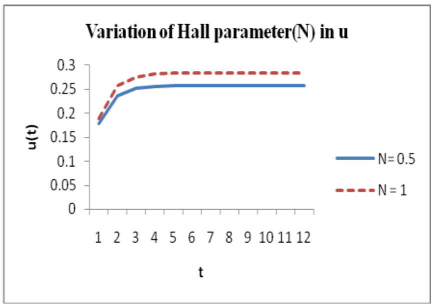

Figure 7

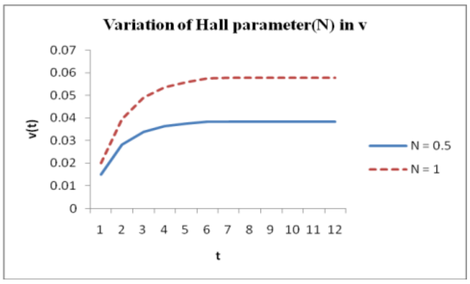

Figure 8

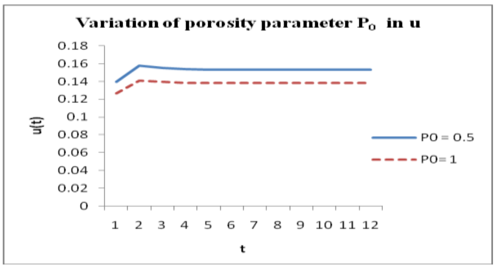

Figure 9

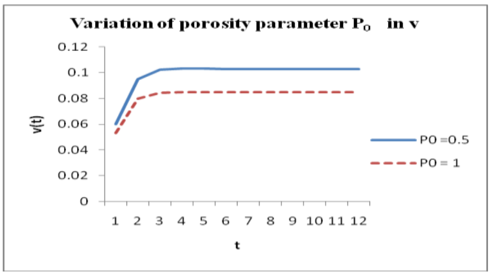

Figure 10

IV. Conclusion

The effect of the Hall parameter N, Hartmann number M and Porosity parameter P_0 in the velocity components u and v are given and the results are shown graphically.

From figures 1,2, 9 & 10, it is clear that both the velocity components decreases with the increase of y and the Porosity parameter. From figure 5 & 6, it is concluded that the velocity components increases with the increase of Hall parameter. From figure 3 The velocity component u decreases and v increases with the increase of Hartmann numberare shown in figure 3 and figure 4 respectively.

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